Confidential

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 23 pages.

MATHE2

INSTRUCTIONS AND INFORMATION

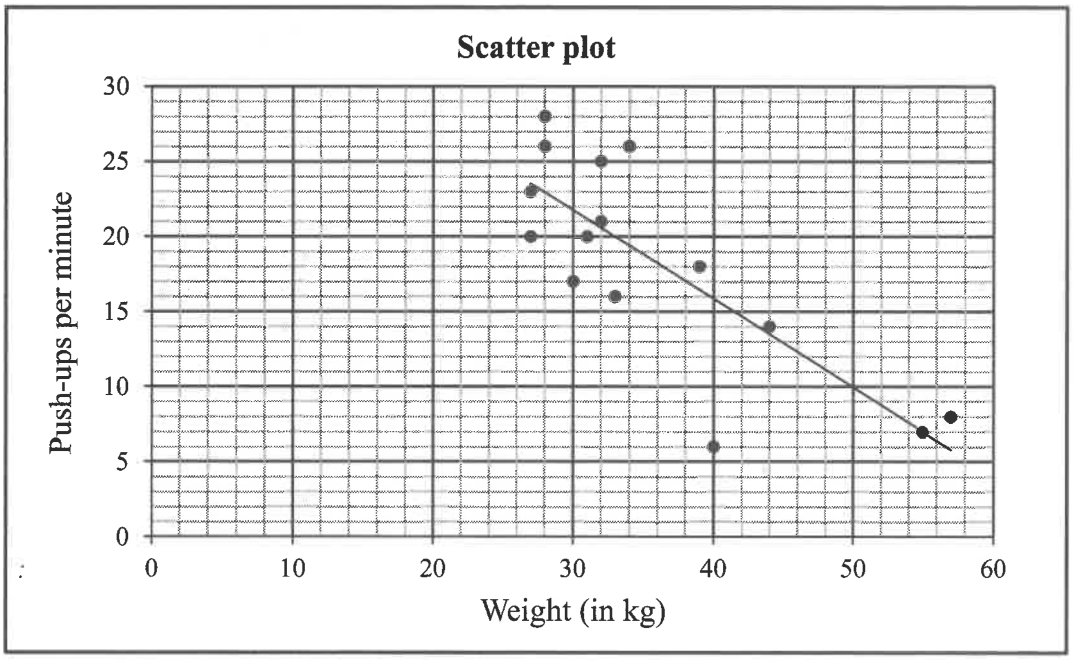
Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

At the beginning of a season, the coach of a junior boys' rugby team recorded the weight (in kg) of the 15 players in his team and the number of push-ups that each player was able to do in one minute. The data is represented in the table and scatter plot below. The least squares regression line for the data is drawn.

Weight (in kg) (x)	34	32	40	27	33	28	27	55	39	44	30	57	28	32	31
Number of push-ups per minute (y)	26	21	6	20	16	26	23	7	18	14	17	8	28	25	20



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Write down the correlation coefficient. (1)

1.3 The coach uses the least squares regression line to set the target for the minimum number of push-ups by each team member according to their weight. Predict the number of push-ups that a member of the team, who weighs 29 kg, should do to meet the target. (2)

1.4 Write down the mean number of push-ups for the given data. (1)

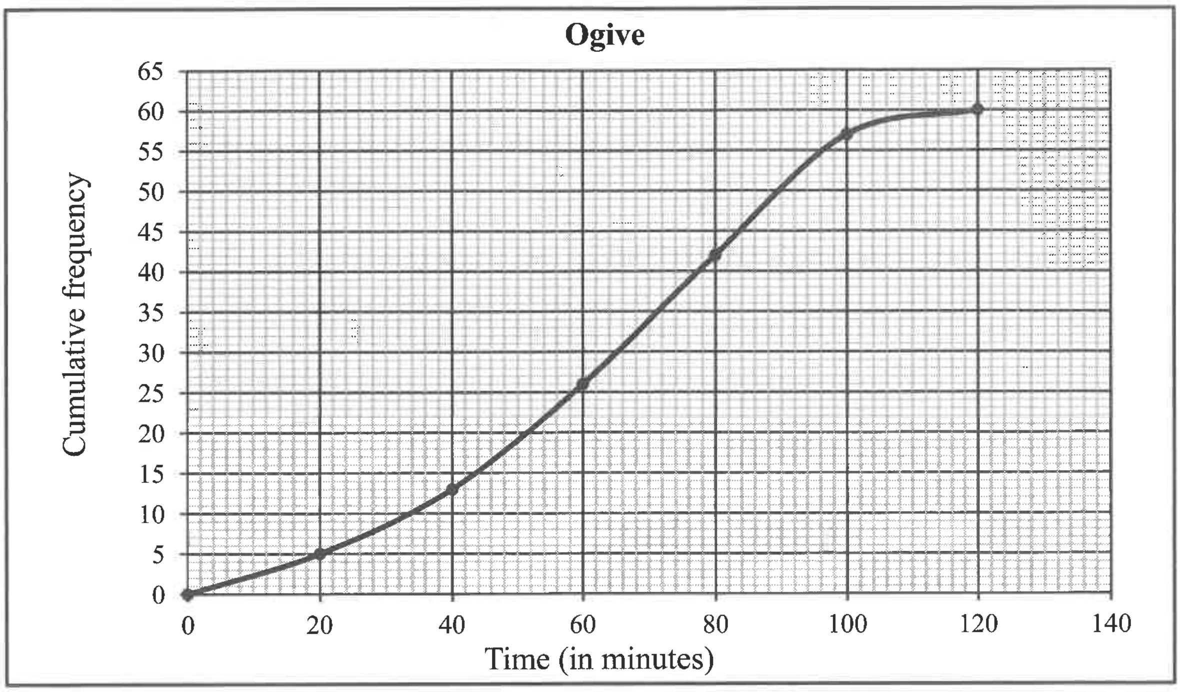
1.5 The players trained hard during the season. At the end of the season, the coach reported that each player was able to do 5 more push-ups per minute than they did at the beginning of the season. How does the increase in the number of push-ups influence the standard deviation of the data? (1)

1.6 At the beginning of the season, the coach used the least squares regression line as the minimum target for a player to aim for. Determine the maximum possible increase in the number of push-ups that a team member must obtain to reach the minimum target. (2)

[10]

QUESTION 2

The cumulative frequency graph (ogive) shows the time taken (in minutes) for 60 employees to travel to work each morning.



2.1 Estimate the median travel time. (1)

2.2 Estimate the lower quartile. (1)

2.3 Estimate the interquartile range. (2)

2.4 The minimum and maximum times taken for an employee to travel to work are 5 and 120 minutes respectively. On the scaled line in the ANSWER BOOK, draw a box and whisker diagram to indicate the distribution of the data as represented in the ogive above. (2)

2.5 The company manager decided that all employees who travel for an hour or more will be allowed to work from home for part of the day. What percentage of the employees will be allowed to work from home for part of the day? (2)

2.6 Employees work 8 hours in a normal working day. The manager decided on the following rule for time to work from home:

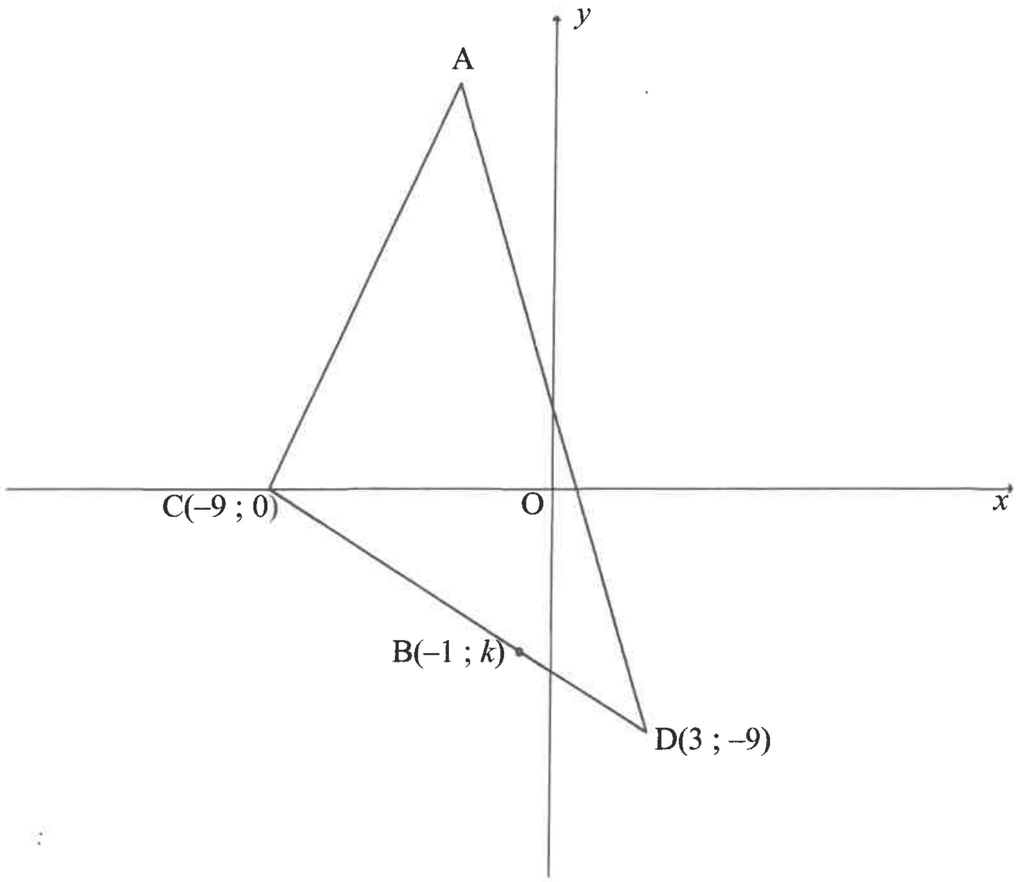
- An employee is allowed to work half an hour from home for each time interval of 20 minutes, or part thereof, above an hour taken to travel to work.

On a certain day, an employee takes 110 minutes to travel to work. Calculate the number of minutes that this employee will be allowed to work from home on this day. (2)

[10]

QUESTION 3

In the diagram below, ΔACD has vertices A, D(3 ; –9) and C(–9 ; 0), where A is a point in the second quadrant. B(–1 ; k) lies on side DC.



3.1 Calculate the gradient of DC. (2)

3.2 Determine the equation of DC in the form $y = mx + c$. (2)

3.3 Show that $k = -6$. (1)

3.4 Calculate the length of DC. (2)

3.5 Calculate the ratio of $\frac{DB}{DC}$. (2)

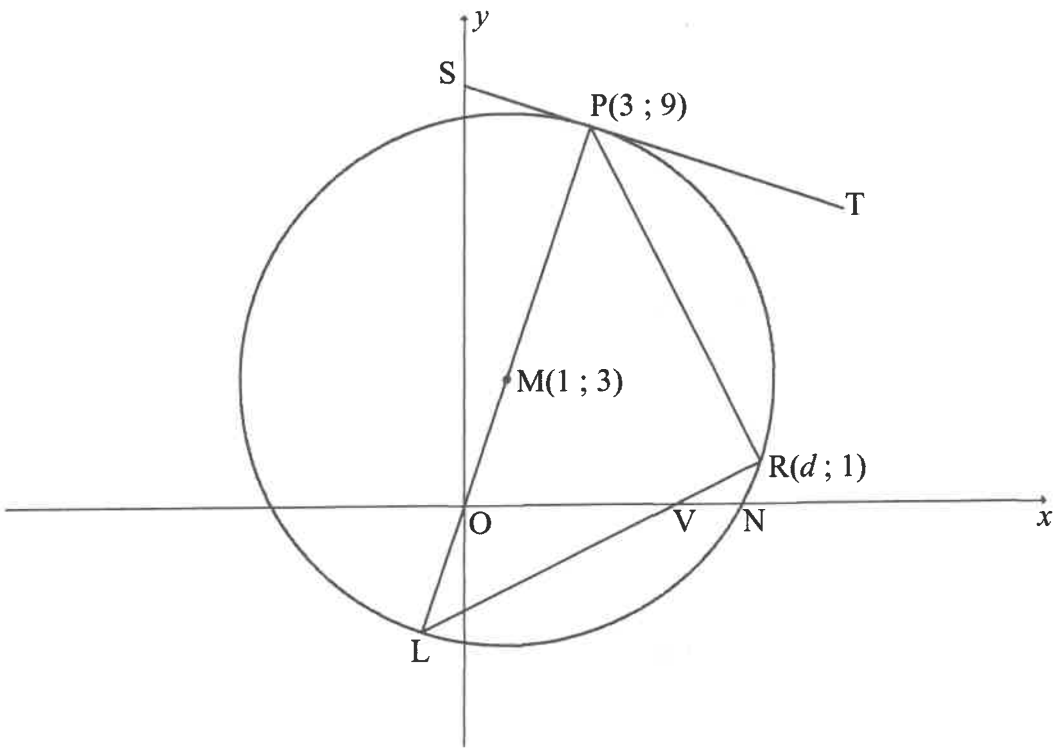
3.6 If M is a point on AD such that AC || MB, calculate the ratio of $\frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD}$. (4)

3.7 If it is further given that the gradient of AD is -4 and the length of AD is $\sqrt{612}$ units, calculate the coordinates of A. (6)

[19]

QUESTION 4

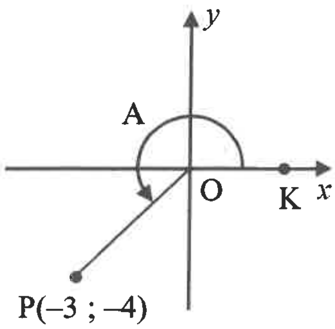
In the diagram, M(1 ; 3) is the centre of the circle. The circle cuts the x-axis at N. ST is a tangent to the circle at P(3 ; 9). R(d ; 1), with $d > 0$, and L lie on the circle. O and V are the x-intercepts of PL and RL respectively.



4.1	Write down the coordinates of L.	(2)
4.2	Determine the equation of tangent ST to the circle at P.	(4)
4.3	Show that the equation of the circle with centre M is $x^2 + y^2 - 2x - 6y - 30 = 0$.	(4)
4.4	Show that $d = 7$.	(2)
4.5	Calculate the size of $\hat{L}$	(5)
4.6	TR is a tangent to the circle at R. Prove that PT $\perp$ RT.	(3)
		[20]

QUESTION 5

5.1 In the diagram, line OP is given with P(–3 ; –4). $K\hat{O}P = A$.



Determine, **without using a calculator**, the value of:

5.1.1 $\cos A$ (2)

5.1.2 $\cos 2A$ (2)

5.1.3 $\sin(A-B)$, if it is further given that $\sin B = \frac{4}{5}$ and $90° < B < 360°$ (4)

5.2 If $\cos\alpha = p$, express the following expression in terms of p:

$$\frac{\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2}$$

(4)
[12]

QUESTION 6

6.1 Given the identity: $\cos(x-y)=\cos x\cos y+\sin x\sin y$

6.1.1 Use the compound angle identity given above to derive a formula for $\cos(x+y)$. (2)

6.1.2 Hence, or otherwise, show that:

$$\frac{\cos(90°-x)\cos y+\sin(-y)\cos(180°+x)}{\cos x\cos(360°+y)+\sin(360°-x)\sin y}=\tan(x+y)$$ (6)

6.2 Given: $f(x)=\sqrt{6\sin^2 x-11\cos(90°+x)+7}$

Solve for x in the interval $x\in(0°\ ;\ 360°)$ if $f(x)=2$. (6)

6.3 Consider the function: $g(x)=\dfrac{4-8\sin^2 x}{3}$

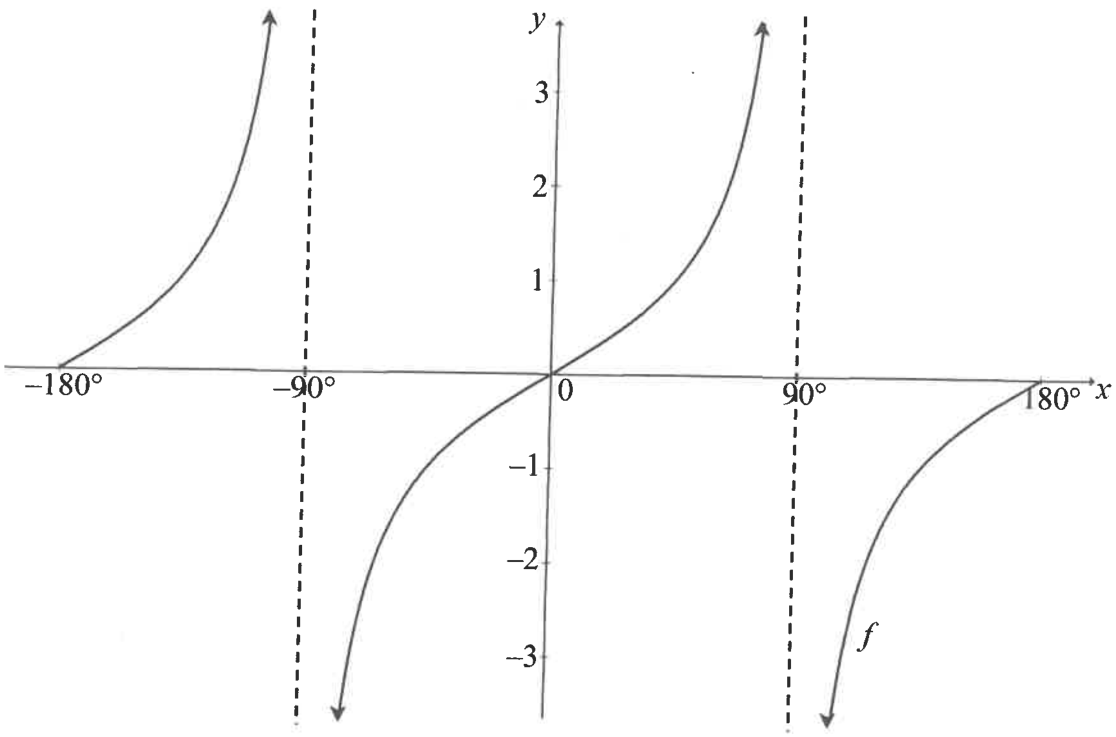
6.3.1 Calculate the maximum value of g. (3)

6.3.2 Write down the smallest possible value of x for which g will have a maximum value in the interval $x\in(0°\ ;\ 360°]$. (1)

[18]

QUESTION 7

In the diagram below, the graph of $f(x) = \tan x$ is drawn for the interval $x \in [-180°\ ;\ 180°]$.



7.1 Write down the equation of the asymptote of f in the interval $x \in [0°\ ;\ 180°]$. (1)

7.2 Write down the values of x in the interval $x \in [-180°\ ;\ 0°]$ for which $f(x) \leq 0$. (2)

7.3 Given: $g(x) = \cos 2x + 1$

7.3.1 Write down the period of g. (1)

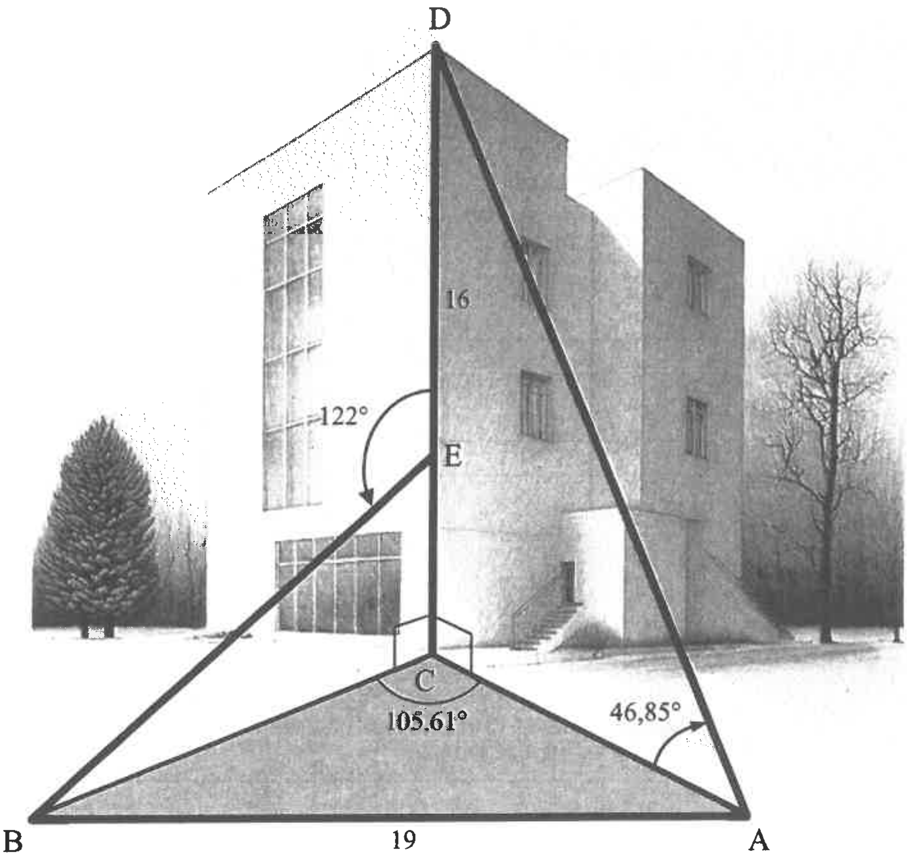
7.3.2 On the grid given in the ANSWER BOOK, draw the graph of $g(x) = \cos 2x + 1$ for the interval $x \in [-180°\ ;\ 180°]$. Clearly show the intercepts with the axes as well as the coordinates of the turning points. (3)

7.4 Use the graphs to determine the general solution of $2\cos^3 x - \sin x = 0$. (4)

[11]

QUESTION 8

In the diagram, C is the foot of a vertical building and D is the top of the same building. The height of the building, CD, is 16 m. Two observers are standing 19 m apart at points A and B, where A, B and C lie in the same horizontal plane. A painter is working at point E on the building. The angle of elevation of D from A is 46,85°. $D\hat{E}B = 122°$ and $B\hat{C}A = 105{,}61°$.



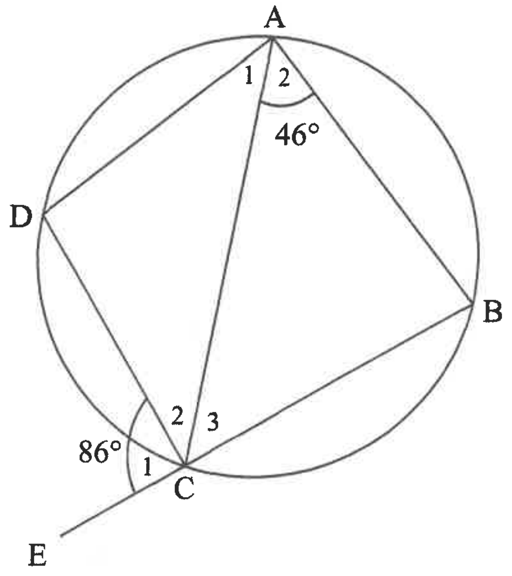
8.1 Calculate the length of AC, the distance between the observer at A and the foot of the building. (2)

8.2 Calculate how far the painter at E is from the top of the building. (7)
[9]

Provide reasons for your statements in QUESTIONS 9, 10 and 11.

QUESTION 9

In the diagram, ABCD is a cyclic quadrilateral. BC is produced to E. AC is drawn.
$\hat{A}_1 = \frac{1}{2}\hat{B}$, $\hat{A}_2 = 46°$ and $\hat{C}_1 = 86°$.

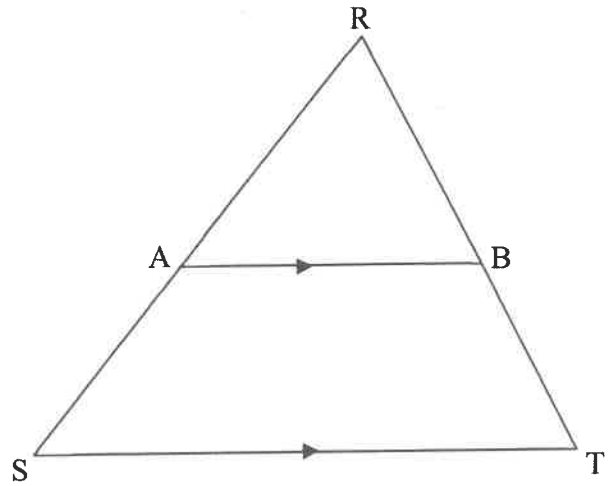


9.1 Calculate, with a reason, the value of $\hat{A}_1$. (2)

9.2 Hence, prove that AD = DC. (4)
[6]

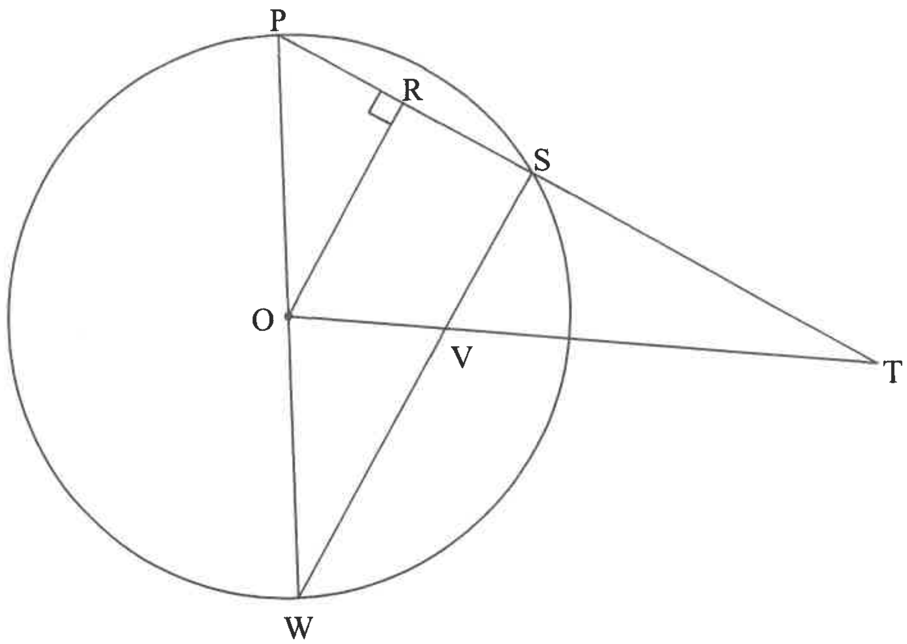
QUESTION 10

10.1 In the diagram, ΔRST is drawn. Line AB intersects RS and RT at A and B respectively such that $AB \parallel ST$.



Prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. $\frac{RA}{AS} = \frac{RB}{BT}$ (6)

10.2 In the diagram, O is the centre of the circle. ΔPWS is drawn with P, W and S on the circle. $OR \perp PS$. PRS is produced to T. SW and OT intersect at V. $OV : OT = 1 : 4$



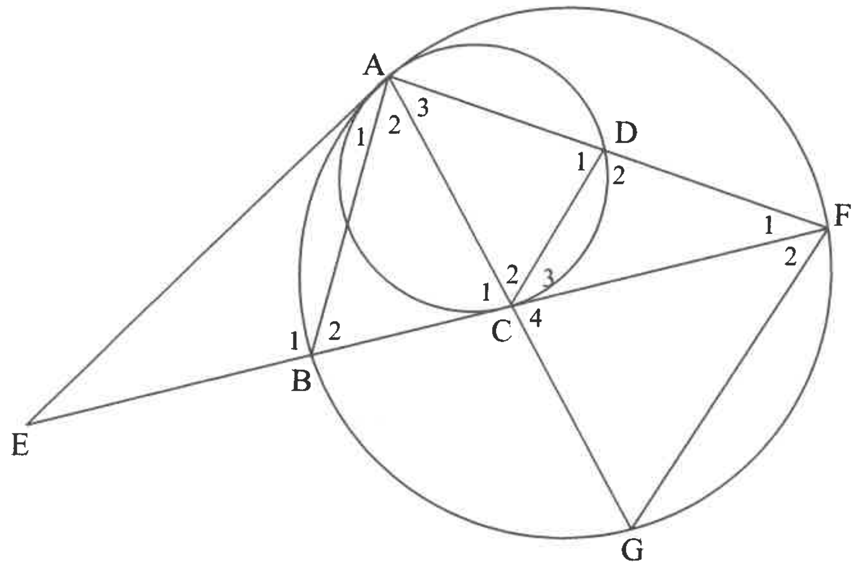
10.2.1 Prove, with reasons, that $OR : WS = 1 : 2$ (5)

10.2.2 Calculate the length of PT if ST = 15 units. (4)

[15]

QUESTION 11

In the diagram, A, B, G and F lie on the larger circle. A smaller circle is drawn to touch the larger circle internally at A. EA is a common tangent to both circles. EBCF is a tangent to the smaller circle at C. AC is produced to G. AF cuts the smaller circle at D. AB, CD and GF are drawn.



11.1 If $E\hat{A}G = x$, determine with reasons, FOUR other angles that are equal to x. (6)

11.2 Prove that AG.AD = AC.AF (4)

11.3 Prove that $\Delta AGF \,|||\, \Delta ABC$ (4)

11.4 Prove that $GF^2 = \dfrac{BC.FC.AF}{AD}$ (6)

[20]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$